Fairness and fairness for neighbors: The difference between the Myerson value and component-wise egalitarian solutions

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Motivation

Many allocation rules for cooperative games are built from two opposite types of principles:

- **Marginalism**: players are rewarded in proportion to what they contribute to coalitions or games (ex: Shapley value),
- **Egalitarianism**: players are rewarded more equally (ex: equal division solution).

Both types have been observed empirically (Aadland and Kolpin, 1998, MSS) and can be mixed (Ju, Borm, Ruys, 2007, SC&W).

It is important to know which properties are satisfied by both types and which permit to distinguish among them.

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A cooperative TU-game on $N = \{1, \ldots, n\}$ is a characteristic function $v : 2^N \rightarrow \mathbb{R}$ such that $v(\emptyset) = 0$.

A TU-game $v$ is zero-normalized if $v(\{i\}) = 0$ for each $i \in N$.

For a TU-games $v$, a real $a \in \mathbb{R}$ and a vector $b \in \mathbb{R}^n$, the game $(av + b)$ is defined as: $\forall S \in 2^N$,

$$(av + b)(S) = av(S) + \sum_{i \in S} b_i.$$ 

An allocation rule is a function $f$ that assigns to each TU-game $v$ on $N$ a payoff vector $f(v) \in \mathbb{R}^n$. 
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The **equal division solution**: \( \forall v, \forall i \in N, \)

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E_i(v) = \frac{v(N)}{n}.
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The **equal surplus division solution**: \( \forall v, \forall i \in N, \)

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Comparable axiomatizations

- Whose player should get a zero-payoff?

van den Brink (2007, JET):
$\Pi$ and $\mathcal{E}$ essentially differ with respect to the null player / nullifying player property.

- Whose player’s deletion does not affect the others’ payoffs?

Kamijo and Kongo (2012, EJOR):
$\Pi$ and $\mathcal{E}$ essentially differ with respect to the null player out / proportional player out property.

We provide similar results for TU-games with limited communication possibilities.
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Communication graphs

An **undirected communication graph** is a pair \((N, L)\) such that \(L\) is a set of links with element \(ij \in L\).

Player \(i\)’s set of **neighbors** is \(L_i = \{j \in N : ij \in L\}\).

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The set of **components** of \((N, L)\) is denoted by \(N/L\).

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An allocation rule on $\mathcal{C}_N$ assigns to each $(v, L) \in \mathcal{C}_N$ a payoff vector $f(v, L) \in \mathbb{R}^n$.

The graph-restricted game $v^L$, (Myerson, 1977, MOR):

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The component-wise egalitarian solution: \( \forall (v, L), \)
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The component-wise egalitarian surplus solution: \( \forall (v, L), \)
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**Component Efficiency.** For each \((v, L) \in \mathcal{C}_N\) and each \(C \in N/L\), it holds

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\sum_{i \in C} f_i(v, L) = v(C).
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**Fairness.** For each \((v, L) \in \mathcal{C}_N\) and each \(ij \in L\), it holds

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f_i(v, L) - f_i(v, L \setminus ij) = f_j(v, L) - f_j(v, L \setminus ij).
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**Proposition [Myerson, 1977, MOR]**

\(\mu\) is the unique allocation rule on \(\mathcal{C}_N\) that satisfies component efficiency and fairness.
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**Proposition [Myerson, 1977, MOR]**

\(\mu\) is the unique allocation rule on \(\mathcal{C}_N\) that satisfies component efficiency and fairness.
Two questions

Q1. Which of his neighbors a player $i$ should care about if one of his links $ij$ is deleted?

— Player $j$ to which he is no longer directly connected and with which he might not be able to communicate?
— Each of his other neighbors to which he is still connected?

Q2. Which notion of efficiency should we consider?

— Efficiency within each component?
— (Overall) efficiency?

Efficiency. For each $(v, L) \in C_N$, it holds that

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**Efficiency.** For each $(v, L) \in \mathcal{C}_N$, it holds that

$$\sum_{i \in N} f_i(v, L) = v(N).$$
Two questions

Q1. Which of his neighbors a player $i$ should care about if one of his links $ij$ is deleted?

— Player $j$ to which he is no longer directly connected and with which he might not be able to communicate?

— Each of his other neighbors to which he is still connected?

Q2. Which notion of efficiency should we consider?

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\[ f_5(v, L) \] and \[ f_8(v, L) \]
Fairness VS Fairness for neighbors

\[ f_5(v, L\setminus 58) \text{ and } f_8(v, L\setminus 58) \]
Fairness: \( f_5(v, L) - f_5(v, L \setminus 58) = f_8(v, L) - f_8(v, L \setminus 58) \)
Player 5 and his neighbors in $(N, L)$
Player 5 and his neighbors in \((N, L\setminus 58)\)
Fairness for neighbors:

\[ f_5(v, L) - f_5(v, L \setminus 58) = f_1(v, L) - f_1(v, L \setminus 58) \]

and

\[ f_5(v, L) - f_5(v, L \setminus 58) = f_2(v, L) - f_2(v, L \setminus 58) \]
Player 8 and his neighbors in \((N, L)\)
Player 8 and his neighbors in \((N, L\setminus 58)\)
Fairness for neighbors:

\[ f_8(v, L) - f_8(v, L\setminus 58) = f_7(v, L) - f_7(v, L\setminus 58) \]

and

\[ f_8(v, L) - f_8(v, L\setminus 58) = f_{10}(v, L) - f_{10}(v, L\setminus 58) \]

and

\[ f_8(v, L) - f_8(v, L\setminus 58) = f_{12}(v, L) - f_{12}(v, L\setminus 58) \]
Fairness for neighbors and the component-wise egalitarian solution

**Fairness for neighbors.** For each \((v, L) \in C_N\), each \(ij \in L\) and each \(k \in L_i \setminus \{j\}\), it holds

\[
f_i(v, L) - f_i(v, L \setminus ij) = f_k(v, L) - f_k(v, L \setminus ij).
\]

**Equal treatment for two-player components.** For each \((v, L) \in C_N\) and each \(\{i, j\} \in N/L\), it holds that

\[
f_i(v, L) = f_j(v, L).
\]

**Proposition**

The component-wise egalitarian solution is the unique allocation rule on \(C_N\) that satisfies component efficiency, fairness for neighbors and equal treatment for two-player components.
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**Proposition**

The **component-wise egalitarian solution** is the unique allocation rule on \(C_N\) that satisfies component efficiency, fairness for neighbors and equal treatment for two-player components.
Other comparable axiomatizations of the Myerson value and the component-wise egalitarian solution:

Slikker (2007, JET) on the class of network games.

**Balanced components.** For each zero-normalized \((v, L)\) and each \(i, j \in N\), it holds that

\[
f_i(v, L) - f_i(v, L \setminus L(C_j)) = f_j(v, L) - f_j(v, L \setminus L(C_i)).
\]

**Proposition [Slikker, 2007, JET]**

The component-wise egalitarian solution is the unique allocation rule on zero-normalized communication situations on \(N\) that satisfies component efficiency and balanced components.
Other comparable axiomatizations of the Myerson value and the component-wise egalitarian solution:
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The component-wise egalitarian solution

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\]

**Proposition [Slikker, 2007, JET]**

The **component-wise egalitarian solution** is the unique allocation rule on zero-normalized communication situations on \(N\) that satisfies component efficiency and balanced components.
**Standardness.** For each \((v, L) \in C_N\), each \(\{i, j\} \in N/L\) and each \(k \in \{i, j\}\), it holds that

\[
f_k(v, L) = v(\{k\}) + \frac{v(\{i, j\}) - v(\{i\}) - v(\{j\})}{2}.
\]

**Covariance.** For each \((v, L) \in C_N\) and each \((av + b, L) \in C_N\), it holds that

\[
f(av + b, L) = af(v, L) + b.
\]
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The component-wise egalitarian surplus solution is the unique allocation rule on $C_N$ that satisfies

- component efficiency, fairness for neighbors and standardness.
- component efficiency, fairness for neighbors, equal treatment for two-player components on zero-normalized communication situations and covariance.
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- component efficiency, fairness for neighbors, equal treatment for two-player components on zero-normalized communication situations and covariance.
- component efficiency, fairness for neighbors and fairness for two-player components.
Can we replace component efficiency by efficiency?

**Component balancedness.** For each \((v, L) \in C_N\) and each \(C \in N/L\), it holds that

\[
\frac{1}{c} \sum_{i \in C} \left( f_i(v, L) - f_i(v_C, L(C)) \right) = \frac{1}{n} \sum_{i \in N} \left( f_i(v, L) - f_i(v_{C_i}, L(C_i)) \right)
\]

**Proposition [van den Brink et al., 2011, WP]**

There exists a unique allocation rule on \(C_N\) that satisfies efficiency, fairness and component balancedness.

For each \((v, L)\), and each \(i \in N\), it assigns the payoff

\[
\psi_i(v, L) = \mu_i(v, L) + \frac{v(N) - v^L(N)}{n}.
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\]
Is fairness for neighbors compatible with efficiency?

**Component payoff equivalence.** For each \((v, L) \in \mathcal{C}_N\), if \(v(C) = v(R)\) for some pair \(\{C, R\} \subseteq N/L\), it holds that

\[
\sum_{i \in C} f_i(v, L) = \sum_{i \in R} f_i(v, L).
\]

**Proposition**

There exists a unique allocation rule on \(\mathcal{C}_N\) that satisfies efficiency, fairness for neighbors, fairness for two-player components, covariance and component payoff equivalence.
Is fairness for neighbors compatible with efficiency?

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**Proposition**

There exists a unique allocation rule on \(C_N\) that satisfies efficiency, fairness for neighbors, fairness for two-player components, covariance and component payoff equivalence.
Double application of the equal surplus division

First application to share $v(N)$ among $C$, $R$, and $T$:

$C$ gets $r_L(C) := v(C) + v(N) - v(C) - v(R) - v(T)$

Second application to share $r_L(C)$ within $C$:

$i \in C$ gets $v(\{i\}) + r_L(C) - \sum_{j\in C} v(\{j\})$
Double application of the equal surplus division

First application to share $v(N)$ among $C, R$ and $T$:

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Double application of the equal surplus division

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Double application of the equal surplus division

First application to share \( v(N) \) among \( C, R \) and \( T \):

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C \text{ gets } r_L(C) := v(C) + v(N) - v(C) - v(R) - v(T)
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Second application to share \( r_L(C) \) within \( C \):

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Double application of the equal surplus division

First application to share $v(N)$ among $C, R$ and $T$:

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Second application to share $r_L(C)$ within $C$:

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\[ \text{Diagram showing nodes and connections for } C, R, T. \]
Double application of the equal surplus division

First application to share $v(N)$ among $C, R$ and $T$:

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Double application of the equal surplus division

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Double application of the equal surplus division

First application to share \( v(N) \) among \( C, R \) and \( T \):

\[
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\]

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Note: The diagrams illustrate the relationships between entities $C$, $R$, and $T$. The numbers represent the values assigned to each entity for the purpose of the division process.
Double application of the equal surplus division

First application to share $v(N)$ among $C, R$ and $T$:

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Double application of the equal surplus division

\[ C \]

First application to share \( v(N) \) among \( C, R \) and \( T \):

- \( C \) gets \( r_L(C) := v(C) - v(R) - v(T) \)

Second application to share \( r_L(C) \) within \( C \):

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2. Fairness and Fairness for neighbors
3. May 15, 2012
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Double application of the equal surplus division

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\[ C \quad R \quad T \]

\[ 3 \quad 6 \]
\[ 2 \quad 5 \]

\[ 9 \quad 12 \]
\[ 8 \quad 11 \]
\[ 10 \]

\[ 1 \quad 4 \quad 7 \]
Double application of the equal surplus division

First application to share \( v(N) \) among \( C, R \) and \( T \):

- \( C \) gets:
  \[
  r_L(C) := v(C) + v(N) - v(C) - v(R) - v(T)
  \]

Second application to share \( r_L(C) \) within \( C \):

- \( i \in C \) gets:
  \[
  v\{i\} + r_L(C) - \sum_{j \in C} v\{j\}
  \]

Diagram:

- \( C \) with nodes 2, 3, 4, 5, 6
- \( R \) with nodes 1, 4, 7
- \( T \) with nodes 8, 9, 10, 11, 12
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Fairness and Fairness for neighbors
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First application to share $v(N)$ among $C$, $R$ and $T$:

$C$ gets $r^L(C) := v(C) + \frac{v(N) - v(C) - v(R) - v(T)}{3}$
Double application of the equal surplus division

Second application to share $r^L(C)$ within $C$:

$$i \in C \text{ gets } v(\{i\}) + \frac{r^L(C) - \sum_{j \in C} v(\{j\})}{4}.$$
A two-step component-wise egalitarian surplus solution

The two-step component-wise egalitarian surplus solution: \( \forall (v, L), \forall C \in N/L, \forall i \in C, \)

\[
ECES_i(v, L) = CES_i(v, L) + \frac{v(N) - v^L(N)}{c \times |N/L|}.
\]

The efficient and fair allocation rule in van den Brink et al. (2011, WP) violates component payoff equivalence.

Is it possible to combine fairness, efficiency and component payoff equivalence?

Claim

There exists a unique allocation rule \( \phi \) on \( C_N \) that satisfies efficiency, fairness, covariance and component payoff equivalence.
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Comparisons (1)

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