An axiomatization of the iterated $h$-index
and applications to sport rankings

Sylvain Béal\textsuperscript{1}, Sylvain Ferrières\textsuperscript{1}, Eric Rémila\textsuperscript{2}, Philippe Solal\textsuperscript{2}

\textsuperscript{1} CRESE EA3190, Univ. Bourgogne Franche-Comté, F25000 Besançon, France
\textsuperscript{2} GATE Lyon Saint-Etienne UMR5824, Université de Saint-Etienne, France

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Motivation

Recent and increasing need to compare/evaluate/rank scholars, research centers, scientific institutions, countries, on the basis of their scientific production.

Few examples/tools:

- Shanghai ranking;
- Web of Science;
- Google Scholar;
- Impact Factor;
- French CNRS ranking for economic journals;
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Objectives/Contributions:

- Presentation of the $h$-index ... and some drawbacks;
- Presentation of a variant which removes one drawback;
- Presentation of an axiomatic characterization;
- Application to sport rankings.
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The scientific production of a scholar is summarized by an ordered list of numbers $x = (x_1, x_2, \ldots, x_n)$ such that

- $n$ is the number of publications of the scholar;
- $x_k, \ k = 1, \ldots, n$, is the number of publications in which publication $k$ is cited, i.e. the number of citations of publication $k$;
- $x_1 \geq x_2 \geq \cdots \geq x_n$, i.e. publications are listed according to their number of citations in decreasing order.

Example: $x = (9, 7, 7, 2, 0)$ means that the scholar has 5 publications, and that the 2nd and 3rd most cited publications have been cited 7 times each.
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The $h$-index is equal to the integer $h$ if the scholar has $h$ publications cited each at least $h$ times, and if any of his/her other publications is cited at most $h$ times.

Drawbacks:

- John F. Nash has won the “Nobel” prize in Economics and had a considerable influence in game theory, but his $h$-index is only 7;
- Self-citations bias;
- The number of citations in a scientific article vary significantly across fields (e.g., in “Molecular Biology & Genetics” compared to Economics);
- Difficulties to compare most of scholars since many of them have (not necessarily small) identical $h$-index. More than 300 scholars registered on REPEC have an $h$-index of 13. —— probably several thousands with equal $h$-index ≤7.
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Definition of the iterated $h$-index

The **iterated $h$-index** or **ih-index** is an empty set $ih(x) = \emptyset$ if $x = \emptyset$ or if $x = (0, \ldots, 0)$, and otherwise the vector

$$ih(x) = (ih_1(x), ih_2(x), \ldots, ih_q(x))$$

such that

- $ih_1(x)$ is the $h$-index of $x$,
- $ih_2(x)$ is the $h$-index of $x$ without the $ih_1(x)$ most-cited publications,
- $ih_3(x)$ is the $h$-index of $x$ without the $ih_1(x) + ih_2(x)$ most-cited publications,
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\[
\begin{align*}
\text{Citations} & \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12} \\
& \text{Publications} \\
& \text{ih}_1(x) = 5 \text{, } \text{ih}_2(x) = 3 \text{, } \text{ih}_3(x) = 1
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$ih(x) = (5, 3, 1, 1, 1)$
Definition of the iterated $h$-index

Close to the multidimensional $h$-index introduced in García-Pérez (2009, Scientometrics).

Two differences:

- if $x = \emptyset$ or if $x = (0, \ldots, 0)$, then the multidimensional $h$-index is equal to $(0)$.
- if $x$ contains non-cited publications, then the multidimensional $h$-index adds a $(0)$ component for each such publication.
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According to the $ih$-index, $x$ is considered as more productive than $y$ if

- $ih_1(x) > ih_1(y)$ or
- $ih_1(x) = ih_1(y)$ and $ih_2(x) > ih_2(y)$, or
- $ih_1(x) = ih_1(y)$, $ih_2(x) = ih_2(y)$ and $ih_3(x) > ih_3(y)$,
- ... or if all comparable components are identical and $ih(x)$ is longer than $ih(y)$.

In other words, if $ih(x)$ lexicographically dominates $ih(y)$.

An alternative interpretation can be provided by means of the Lorenz domination.
According to the *ih*-index, *x* is considered as **more productive** than *y* if

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An alternative interpretation can be provided by means of the **Lorenz** domination.
A **generalized index** is a function $f$ which assigns to each vector $x$ another vector $f(x) = (f_1(x), \ldots, f_{qx}(x))$ satisfying:

- **No publication/no citation** benchmark: if $x = \emptyset$ or if $x = (0, \ldots, 0)$, then $f(x) = \emptyset$;

- **Monotonicity**: if $x$ is longer than $y$ and greater than or equal to $y$ (coordinate by coordinate), then $f(x)$ is longer than $y$ and greater than or equal to $f(y)$ (coordinate by coordinate).
Examples (for cases when $x_1 > 0$)

\[ f(x) = i h(x), \quad \text{i.e. } i h\text{-index} \]

\[ f(x) = x, \quad \text{i.e. } x \text{ itself} \]

\[ f(x) = \left( \frac{1}{k} \sum_{i=1}^{k} x_i \right)_{k \in \{1, \ldots, n\}}, \quad \text{i.e. the average numbers of citations} \]

\[ f(x) = (h(x)), \quad \text{i.e. the unique component is the } h\text{-index} \]

\[ f(x) = \left( \sqrt{\sum_{i=1}^{n} x_i^2} \right), \quad \text{i.e. the Euclidean index (Perry and Reny, AER, 2016)} \]

\[ f(x) = \left( \sum_{i=1}^{n} x_i \right), \quad \text{i.e. total number of citations} \]

\[ f(x) = (n), \quad \text{i.e. number of publications} \]

\[ f(x) = \left( \max\{i = 1, \ldots, n : x_i > 0\} \right), \quad \text{i.e. number of cited publications} \]

\[ f(x) = (x_1), \quad \text{i.e. number of citations of the most cited publication} \]
Examples (for cases when $x_1 > 0$)

\[ f(x) = ih(x), \quad \text{i.e. } ih\text{-index} \]
\[ f(x) = x, \quad \text{i.e. } x \text{ itself} \]
\[ f(x) = \left( \frac{1}{k} \sum_{i=1}^{k} x_i \right)_{k \in \{1, \ldots, n\}}, \quad \text{i.e. the average numbers of citations} \]
\[ f(x) = (h(x)), \quad \text{i.e. the unique component is the } h\text{-index} \]
\[ f(x) = \left( \sqrt{\sum_{i=1}^{n} x_i^2} \right), \quad \text{i.e. the Euclidean index} \quad (\text{Perry and Reny, AER, 2016}) \]
\[ f(x) = \left( \sum_{i=1}^{n} x_i \right), \quad \text{i.e. total number of citations} \]
\[ f(x) = (n), \quad \text{i.e. number of publications} \]
\[ f(x) = (\max\{i = 1, \ldots, n : x_i > 0\}), \quad \text{i.e. number of cited publications} \]
\[ f(x) = (x_1), \quad \text{i.e. number of citations of the most cited publication} \]
Examples (for cases when $x_1 > 0$)

$f(x) = ih(x)$, i.e. $ih$-index

$f(x) = x$, i.e. $x$ itself

$f(x) = \left(\frac{1}{k} \sum_{i=1}^{k} x_i\right)_{k \in \{1,...,n\}}$, i.e. the average numbers of citations

$f(x) = (h(x))$, i.e. the unique component is the $h$-index

$f(x) = (\sqrt{\sum_{i=1}^{n} x_i^2})$, i.e. the Euclidean index (Perry and Reny, AER, 2016)

$f(x) = (\sum_{i=1}^{n} x_i)$, i.e. total number of citations

$f(x) = (n)$, i.e. number of publications

$f(x) = (\max\{i = 1,\ldots,n : x_i > 0\})$, i.e. number of cited publications

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**Axioms**

**One citation case.** If the total of citations in $x$ is one, then $f(x) = (1)$.

Example: if $x = (1, 0, \ldots, 0)$, then $f(x) = (1)$.

**Homogeneity.** If $y$ is obtained from $x$ by multiplying by $c \in \mathbb{N}$ the number of citations of each publication, and then by multiplying by $c$ the number of publications, then $f(y)$ is equal to $c$ times $f(x)$.

Example: let $x = (4, 2, 1)$, $c = 3$, and $y = (12, 12, 12, 6, 6, 6, 3, 3, 3)$. If $f(x) = (3, 2)$ has already been computed, then $f(y) = 3 \times f(x) = (9, 3)$. 
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**Independence of superfluous citations.** Adding extra citations to the $f_1(x)$ most-cited publications does not alter the generalized index.

Example: if $x = (7, 4, 2, 1)$, $f_1(x) = 2$ and $y = (9, 5, 2, 1)$ then $f(y) = f(x)$.

**Independence of irrelevant publications.** Adding publications that have at most $f_k(x)$ publications does not alter the first $k$ components of the generalized index.

Example: if $x = (7, 4, 2, 1)$, $f_2(x) = 2$ and $y = (7, 4, 2, 2, 2, 2, 2, 1, 1, 0, 0, 0)$ then $f_1(y) = f_1(x)$ and $f_2(y) = f_2(x)$.
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**Consistency.** If $y$ is obtained from $x$ by removing the $f_1(x) + \cdots + f_k(x)$ most-cited publications, then $f(y)$ coincides with components $k + 1, k + 2, \ldots$ of $f(x)$.

Example: (for the simpler case $k = 1$) if $x = (7, 6, 4, 2, 1)$, $f(x) = (2, 2, 1)$ and $y = (4, 2, 1)$, then $f(y) = (f_2(x), f_3(x)) = (2, 1)$. 
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Characterizations

**Theorem**

The *ih*-index is the unique generalized index that satisfies One citation case, Homogeneity, Independence of superfluous citations, Independence of irrelevant publications, and Consistency.
Characterizations

The (generalized) $h$-index $f(x) = (h(x))$ satisfies all axioms except Consistency.

Strong independence of irrelevant publications. Adding publications that have at most $f_1(x)$ citations does not alter the generalized index.

**Theorem**

The (generalized) $h$-index $f(x) = (h(x))$ is the unique generalized index that satisfies One citation case, Homogeneity, Independence of superfluous citations, and Strong independence of irrelevant publications.
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The (generalized) $h$-index $f(x) = (h(x))$ is the unique generalized index that satisfies One citation case, Homogeneity, Independence of superfluous citations, and Strong independence of irrelevant publications.
Correspondence between scientific productions and sports records:

- publications $\leftrightarrow$ matches won by a player/team.
- citations of a publication $\leftrightarrow$ matches won by the defeated player/team.

Example: ATP 2015 season of N. Djokovic. 82 matches won, among which 6 times against A. Murray, who has won 63 matches (more than any other player defeated by Djokovic).

$\implies$ Djokovic_2015 = $(63, 63, 63, 63, 63, 63, 61, 61, 61, 61, 60, 60, 60, 60, 60, 57, 57, 57, 57, 57, 57, 54, 54, 54, 53, 53, 53, 51, 51, 46, 45, 45, 45, 45, 43, 43, 41, 41, 37, 37, 37, 37, 35, 35, 35, 35, 34, 34, 33, 33, 32, 32, 32, 32, 31, 31, 31, 31, 30, 30, 30, 30, 30, 30, 30, 27, 27, 27, 27, 27, 27, 26, 26, 25, 24, 22, 22, 21, 20, 20, 20, 19, 18, 15, 14, 14, 13, 10, 8, 4)$. 
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$(63, 63, 63, 63, 63, 63, 61, 61, 61, 61, 60, 60, 60, 60, 60, 57, 57, 57, 57, 57, 54, 54, 54, 53, 53, 53, 51, 51, 46, 45, 45, 45, 45, 43, 43, 41, 41, 37, 37, 37, 37, 37, 35, 35, 35, 35, 34, 34, 33, 33, 32, 32, 32, 32, 31, 31, 31, 31, 30, 30, 30, 30, 30, 27, 27, 27, 27, 27, 26, 26, 25, 24, 22, 22, 21, 20, 20, 20, 19, 18, 15, 14, 14, 13, 10, 8, 4)$. 
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$$34, 33, 33, 32, 32, 32, 32, 31, 31, 31, 31, 30, 30, 30, 30, 30, 27, 27, 27, 27, 27, 27,$$ 
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## 2014–2015 French League 1 football

<table>
<thead>
<tr>
<th>Team</th>
<th>$ih$-index (rank)</th>
<th>League pts (rank)</th>
<th>Diff.</th>
<th>TV rights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paris</td>
<td>(12, 8, 4) (1)</td>
<td>83 (1)</td>
<td>=</td>
<td>15 714 696 €</td>
</tr>
<tr>
<td>Lyon</td>
<td>(12, 8, 2) (2)</td>
<td>75 (2)</td>
<td>=</td>
<td>13 663 055 €</td>
</tr>
<tr>
<td>Marseille</td>
<td>(12, 7, 2) (3)</td>
<td>69 (4)</td>
<td>▲1</td>
<td>10 323 682 €</td>
</tr>
<tr>
<td>Monaco</td>
<td>(12, 7, 1) (4)</td>
<td>71 (3)</td>
<td>▼1</td>
<td>11 873 326 €</td>
</tr>
<tr>
<td>Saint-Etienne</td>
<td>(12, 7) (5)</td>
<td>69 (5)</td>
<td>=</td>
<td>8 970 472 €</td>
</tr>
<tr>
<td>Bordeaux</td>
<td>(12, 5) (6)</td>
<td>63 (6)</td>
<td>=</td>
<td>7 802 783 €</td>
</tr>
<tr>
<td>Guingamp</td>
<td>(12, 3) (7)</td>
<td>49 (10)</td>
<td>▲3</td>
<td>4 452 497 €</td>
</tr>
<tr>
<td>Montpellier</td>
<td>(11, 5) (8)</td>
<td>56 (7)</td>
<td>▼1</td>
<td>6 787 876 €</td>
</tr>
<tr>
<td>Lille</td>
<td>(11, 5) (9)</td>
<td>56 (8)</td>
<td>▼1</td>
<td>5 893 011 €</td>
</tr>
<tr>
<td>Nice</td>
<td>(11, 2) (10)</td>
<td>48 (11)</td>
<td>▲1</td>
<td>3 874 109 €</td>
</tr>
<tr>
<td>Caen</td>
<td>(11, 1) (11)</td>
<td>46 (13)</td>
<td>▲2</td>
<td>2 924 680 €</td>
</tr>
<tr>
<td>Bastia</td>
<td>(11, 1) (12)</td>
<td>47 (12)</td>
<td>=</td>
<td>3 372 112 €</td>
</tr>
<tr>
<td>Reims</td>
<td>(11, 1) (13)</td>
<td>44 (15)</td>
<td>▲2</td>
<td>2 215 336 €</td>
</tr>
<tr>
<td>Toulouse</td>
<td>(11, 1) (14)</td>
<td>42 (17)</td>
<td>▲3</td>
<td>1 669 686 €</td>
</tr>
<tr>
<td>Rennes</td>
<td>(10, 3) (15)</td>
<td>50 (9)</td>
<td>▼6</td>
<td>5 129 102 €</td>
</tr>
<tr>
<td>Nantes</td>
<td>(10, 1) (16)</td>
<td>45 (14)</td>
<td>▼2</td>
<td>2 542 725 €</td>
</tr>
<tr>
<td>Lorient</td>
<td>(9, 3) (17)</td>
<td>43 (16)</td>
<td>▼1</td>
<td>1 920 685 €</td>
</tr>
<tr>
<td>Evian</td>
<td>(7, 4) (18)</td>
<td>37 (18)</td>
<td>=</td>
<td>0 €</td>
</tr>
<tr>
<td>Lens</td>
<td>(7) (19)</td>
<td>29 (20)</td>
<td>▲1</td>
<td>0 €</td>
</tr>
<tr>
<td>Metz</td>
<td>(7) (20)</td>
<td>30 (19)</td>
<td>▼1</td>
<td>0 €</td>
</tr>
</tbody>
</table>

Béal, Ferrières, Rémiła, Solal

10th seminar day – OSGAD, November 8th, 2016
Classical $h$-index not very relevant: too many teams with the same $h$-index.

Tie/draw not taken into account by the $ih$-index $\Rightarrow$ incentive to win comparable to the rule giving three points for a win.

Same 5 teams qualified for European competitions (only 2 teams swap positions); same 3 teams relegated to the second division.

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Is the $ih$-index close or not to the official ATP ranking?

<table>
<thead>
<tr>
<th>Player</th>
<th>$ih$-index (rank)</th>
<th>ATP points (rank)</th>
<th>Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Novak Djokovic</td>
<td>(37, 26, 14, 4, 1)</td>
<td>16 585 (1)</td>
<td>=</td>
</tr>
<tr>
<td>Andy Murray</td>
<td>(31, 23, 6, 2, 1)</td>
<td>8 945 (2)</td>
<td>=</td>
</tr>
<tr>
<td>Roger Federer</td>
<td>(31, 18, 11, 1)</td>
<td>8 265 (3)</td>
<td>=</td>
</tr>
<tr>
<td>Stan Wawrinka</td>
<td>(30, 16, 6, 2)</td>
<td>6 865 (4)</td>
<td>=</td>
</tr>
<tr>
<td>Rafael Nadal</td>
<td>(28, 19, 9, 3)</td>
<td>5 230 (5)</td>
<td>=</td>
</tr>
<tr>
<td>Tomas Berdych</td>
<td>(27, 20, 7, 2, 1)</td>
<td>4 620 (6)</td>
<td>=</td>
</tr>
<tr>
<td>Key Nishikori</td>
<td>(26, 16, 7, 2)</td>
<td>4 235 (8)</td>
<td>▲1</td>
</tr>
<tr>
<td>John Isner</td>
<td>(25, 13, 5, 2)</td>
<td>2 495 (11)</td>
<td>▲3</td>
</tr>
<tr>
<td>Richard Gasquet</td>
<td>(25, 13, 4, 1)</td>
<td>2 850 (9)</td>
<td>=</td>
</tr>
<tr>
<td>David Ferrer</td>
<td>(24, 18, 9, 2)</td>
<td>4 305 (7)</td>
<td>▼3</td>
</tr>
<tr>
<td>Gilles Simon</td>
<td>(24, 12, 4)</td>
<td>2 145 (15)</td>
<td>▲4</td>
</tr>
<tr>
<td>Kevin Anderson</td>
<td>(23, 15, 4, 2, 2)</td>
<td>2 475 (12)</td>
<td>=</td>
</tr>
<tr>
<td>Roberto Bautista Agut</td>
<td>(22, 12, 4, 2)</td>
<td>1 480 (25)</td>
<td>▲12</td>
</tr>
<tr>
<td>Ivo Karlovic</td>
<td>(22, 12, 3)</td>
<td>1 485 (23)</td>
<td>▲9</td>
</tr>
<tr>
<td>Dominic Thiem</td>
<td>(22, 12, 2)</td>
<td>1 600 (20)</td>
<td>▲5</td>
</tr>
<tr>
<td>Gaël Monfils</td>
<td>(21, 10, 2)</td>
<td>1 485 (24)</td>
<td>▲8</td>
</tr>
<tr>
<td>Jo-Wilfried Tsonga</td>
<td>(21, 9, 2)</td>
<td>2 635 (10)</td>
<td>▼7</td>
</tr>
<tr>
<td>Milos Raonic</td>
<td>(21, 9, 2)</td>
<td>2 170 (14)</td>
<td>▼4</td>
</tr>
<tr>
<td>Viktor Troicki</td>
<td>(21, 8, 4)</td>
<td>1 487 (22)</td>
<td>▲3</td>
</tr>
<tr>
<td>Feliciano Lopez</td>
<td>(21, 8, 2, 1)</td>
<td>1 690 (17)</td>
<td>▼3</td>
</tr>
</tbody>
</table>
9 (47 resp.) players are common to both both top 10 (50 resp.).

Very few equal $i_h$-indices for top 100 players.

Notable differences in rankings. Four possible explanations:

1. injured players with good performances (Tsonga): higher ATP that $i_h$-index ranking,
2. players with points coming from the second (challenger tour) or third (ITF tour) level tournaments (Paire): higher ATP that $i_h$-index ranking,
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## Example for the Eastern conference

<table>
<thead>
<tr>
<th>Team</th>
<th>(ih)-index (rank)</th>
<th>Winning % (rank)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toronto Raptors</td>
<td>(35, 17, 4) (1)</td>
<td>0.683 (2)</td>
<td>▲1</td>
</tr>
<tr>
<td>Cleveland Cavaliers</td>
<td>(33, 18, 6) (2)</td>
<td>0.695 (1)</td>
<td>▼1</td>
</tr>
<tr>
<td>Atlanta Hawks</td>
<td>(33, 12, 3) (3)</td>
<td>0.585 (4)</td>
<td>▲1</td>
</tr>
<tr>
<td>Miami Heats</td>
<td>(33, 12, 3) (4)</td>
<td>0.585 (3)</td>
<td>▼1</td>
</tr>
<tr>
<td>Boston Celtics</td>
<td>(33, 11, 3) (5)</td>
<td>0.585 (5)</td>
<td>=</td>
</tr>
<tr>
<td>Charlotte Hornets</td>
<td>(32, 12, 4) (6)</td>
<td>0.585 (6)</td>
<td>=</td>
</tr>
<tr>
<td>Detroit Pistons</td>
<td>(32, 10, 2) (7)</td>
<td>0.537 (8)</td>
<td>▲1</td>
</tr>
<tr>
<td>Indiana Pacers</td>
<td>(31, 11, 3) (8)</td>
<td>0.549 (7)</td>
<td>▼1</td>
</tr>
<tr>
<td>Chicago Bulls</td>
<td>(31, 10, 1) (9)</td>
<td>0.512 (9)</td>
<td>=</td>
</tr>
<tr>
<td>Washington Wizards</td>
<td>(30, 10, 1) (10)</td>
<td>0.500 (10)</td>
<td>=</td>
</tr>
<tr>
<td>Orlando Magic</td>
<td>(27, 8) (11)</td>
<td>0.427 (11)</td>
<td>=</td>
</tr>
<tr>
<td>Milwaukee Bucks</td>
<td>(24, 9) (12)</td>
<td>0.402 (12)</td>
<td>=</td>
</tr>
<tr>
<td>New York Knicks</td>
<td>(23, 9) (13)</td>
<td>0.390 (13)</td>
<td>=</td>
</tr>
<tr>
<td>Brooklyn Nets</td>
<td>(19, 2) (14)</td>
<td>0.256 (14)</td>
<td>=</td>
</tr>
<tr>
<td>Philadelphia 76ers</td>
<td>(10) (15)</td>
<td>0.122 (15)</td>
<td>=</td>
</tr>
</tbody>
</table>
Same top 16 teams qualified for the playoff.

Small differences $\implies$ switches in positions in the bracket (can cancel the home-court advantage).

No difference in ranking for the bottom 14 teams $\implies$ no changes regarding the chances to get the best position in the next NBA draft.
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